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Investigation of Effects of Viscoelastic Boundary Supports on Transient Sound Radiated from a Rectangular Plate by Modal Strain Energy Method

Zi-Jie Fan* and Kwang-Joon Kim*

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This work considers analysis of transient sound radiation from an impact-excited rectangular plate with viscoelastic boundary supports based on the Modal Strain Energy (MSE) method. Vibration of the plate is approximated by double infinite series in the spatial coordinates. Each term of the series is constructed with vibration modes of beams having the same boundary conditions as the considered plate, multiplied by a time dependent function. Modal loss factor of each mode is obtained by the MSE method. The sound pressure for impact excitations is obtained in the time and frequency domain by numerical integration of the Rayleigh integral. Then effects of width of the viscoelastic boundary supports on the vibration response and the radiated sound pressure are investigated. It is shown that there is an optimum width of the support.

Key Words: Transient Sound Radiation, Viscoelastic Boundary Supports, Modal Strain Energy Method

1. Introduction

Viscoelastic surface damping treatments have been used for many years to reduce vibration and noise of structures especially for beam and plate-like structures. Such surface damping treatments have been shown to be effective in vibration and noise control. In many cases, however, it is not always possible to implement them in real situations. In such cases damping treatment at the boundary supports can be an alternative solution (MacBain, 1975 and Harris, 1988).

Recently, Kang and Kim (1996a) proposed a systematic method to estimate modal properties of beams and plates with viscoelastic boundary supports. For beam vibrations, the viscoelastic support regions are described analytically in terms of frequency-dependent complex stiffness and, then, characteristic equations of the beam structure supported at its ends by springs with such complex stiffness are derived. Natural frequencies and modal loss factors of the assembled beam system are obtained by solving transcendental characteristic equations numerically. Similar approach can be applied to plates with viscoelastic boundary supports.

A number of studies have been made on acoustic radiation from impact-excited plates. Strasberg (1948) calculated acoustic power radiated from a periodically struck diaphragm with fixed boundary support in the frequency domain. Formulation of the radiation problem was based on the results of Lax (1944), who investigated radiation loading on a circular clamped plate using the Rayleigh integral equation. Several other studies on the acoustic radiation from impacted plates have led to empirical relationships between acoustic pressures and plate vibration responses. Nagayama et al. (1981) reported on transient sound radiated from a clamped circular plate excited by an impulsive plane wave. Akay et al. (1983) investigated sound radiation from an impacted clamped circular plate in an infinite

* Center for Noise and Vibration Control Department of Mechanical Engineering KAIST

baffle. Teon et al.(1987) evaluated transient sound radiation from a clamped circular plate with viscoelastic layers.

Kang (1996) showed that there exists an optimum width of the viscoelastic boundary supports to obtain maximum modal damping for a few lowest modes. In this paper, transient sound radiation from rectangular baffled plates with viscoelastic boundary support at each edge subject to an impact load is examined to observe whether there exists an optimum support width. Vibration of the plate is approximated by double infinite series in the spatial coordinates. Each term of the series consists of a product of two vibration modes of beams, having the same boundary conditions as the considered plate, multiplied by a time dependent function. Modal loss factor of each mode is obtained by the MSE method. Sound pressure radiated from the plate is obtained in the time and frequency domain by numerical integration of the Rayleigh integral.

2. Analysis

2.1 Equivalent system for a rectangular plate with viscoelastic boundary supports

Figure 1(a) shows a rectangular plate of length a, width b and thickness h with uniform viscoelastic boundary supports along the four edges. Each viscoelastic support has thickness H and width L, of which the material property related with deformation is given by complex modulus $E^* = E(1+j\eta)$ where η is called loss factor of the material.

According to Kang and Kim's work (1996a, b), an equivalent system for the rectangular plate with viscoelastic boundary supports can be obtained by using stiffness parameters at the boundaries as shown in Fig. 1(b). Stiffness parameters K_{11} and K_{22} are frequency dependent complex quantities as the moduli of the viscoelastic support are inherently frequency dependent and complex.

2.2 Equation of motion of the plate and associated boundary conditions

The equation of motion of a thin, elastic, homogeneous, and isotropic plate subjected to an excitation force can be written as

$$D\nabla^4 w(x, y, t) + \rho h \partial^2 w(x, y, t)/\partial t^2 = q(x, y, t) \tag{1}$$

where

$D = Eh^3/12(1-\nu^2)$: flexural rigidity of the plate

$\nabla^4 = \partial^4/\partial x^4 + \partial^4/\partial x^2 \partial y^2 + \partial^4/\partial y^4$: Bi-Laplacian

E: Young's modulus

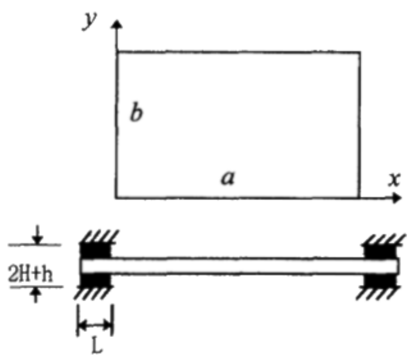
w: transverse deflection

ν: Poisson's ratio

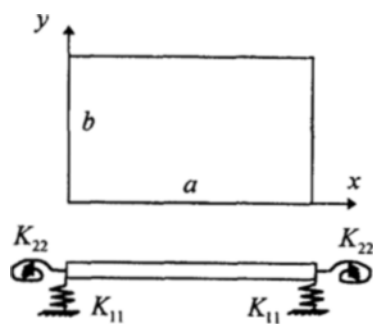
ρ: mass per unit area

q: excitation force per unit area.

If the plate is subjected to an impact force $F(t)$ concentrated at a point (x_0, y_0), the excitation force $q(x, y, t)$ can be expressed as a spatial Dirac delta function as follows:



(a) Dimension of viscoelastic supports



(b) Modeling of viscoelastic supports

Fig. 1 A rectangular plate with viscoelastic boundary supports and its equivalent system

$$q(x, y, t) = F(t)\,\delta(x-x_0)\,\delta(y-y_0) \quad (2)$$

Two boundary conditions must be prescribed on each of the four edges of the rectangular plate; $x=0$, $x=a$, $y=0$, $y=b$ (see Fig. 1(b)). The boundary conditions along line $x=0$ are given by

$$V_x = D(\partial^3 w/\partial x^3 + (2-\nu)\,\partial^3 w/\partial x \partial y^2)_{x=0} = -K_{11}(w)_{x=0} \quad (3a)$$

$$M_x = D(\partial^2 w/\partial x^2 + \nu \partial^2 w/\partial y^2)_{x=0} = K_{22}(\partial w/\partial x)_{x=0} \quad (3b)$$

where V_x and M_x are the shear force and the bending moment per unit length along the vertical edge respectively. The boundary conditions for the remaining viscoelastic supports can be described in a similar manner.

Zero initial conditions are assumed before an impact is provided, i. e.,

$$w\,|_{t=0} = 0 \text{ and } \partial w/\partial t\,|_{t=0} = 0 \quad (4)$$

2.3 Solution method

Leissa(1969) suggested that an approximate solution to Eq. (1) can be expressed by

$$w(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}(x, y)\, g_{mn}(t) \quad (5)$$

where $g_{mn}(t)$ is a time-dependent function and $W_{mn}(x, y)$ a mode shape function of the undamped rectangular plate system given by $W_{mn}(x, y) = X_m(x)\,Y_n(y)$. $X_m(x)$ and $Y_n(y)$ and can be taken from the mode shape functions in the x and y directions, respectively of the associated undamped beams with the same boundary conditions as the considered plate:

$$X_m(x) = A_m \sin \beta_m \frac{x}{a} + B_m \cos \beta_m \frac{x}{a} + C_m \sinh \beta_m \frac{x}{a} + D_m \cosh \beta_m \frac{x}{a} \quad (6a)$$

$$Y_n(y) = A_n \sin \beta_n \frac{y}{b} + B_n \cos \beta_n \frac{y}{b} + C_n \sinh \beta_n \frac{y}{b} + D_n \cosh \beta_n \frac{y}{b} \quad (6b)$$

Substituting Eqs. (2), (5), and (6) into Eq. (1) leads to

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\{ X_m Y_n[D g_{mn}\Big(\Big(\frac{\beta_m}{a}\Big)^4 + \Big(\frac{\beta_n}{b}\Big)^4\Big) + \rho h \ddot{g}_{mn}] + 2D X_m'' Y_n'' g_{mn}\} = F(t)\,\delta(x-x_0)\,\delta(y-y_0) \quad (7)$$

where the prime and dot denote differentiation with respect to the spatial and time coordinates respectively. Multiplying both sides of Eq. (7) by the mode function $X_m(x)\,Y_n(y)$ and integrating over the plate makes the equation decoupled due to the orthogonality of $X_m(x)$ and $Y_n(y)$ as follows:

$$M_{mn}\ddot{g}_{mn} + K_{mn} g_{mn} = X_m(x_0)\,Y_n(y_0)\,F(t) \quad (m, n = 1, 2, 3, \cdots) \quad (8)$$

where

$$M_{mn} = \rho h \int_0^a \int_0^b X_m^2 Y_n^2\, dxdy \quad (9)$$

$$K_{mn} = D \int_0^a \int_0^b \Big[\Big(\Big(\frac{\beta_m}{a}\Big)^4 + \Big(\frac{\beta_n}{b}\Big)^4\Big) X_m^2 Y_n^2 + 2 X_m Y_n X_m'' Y_n''\Big] dxdy \quad (10)$$

Defining undamped natural frequency ω_{mn} by

$$\omega_{mn} = \Big(\frac{K_{mn}}{M_{mn}}\Big)^{1/2} \quad (11)$$

Eq. (8) can be rewritten as

$$\ddot{g}_{mn} + \omega_{mn}^2 g_{mn} = \frac{1}{M_{mn}} X_m(x_0)\,Y_n(y_0)\,F(t) \quad (m, n = 1, 2, 3, \cdots) \quad (12)$$

In the MSE method, it is assumed that vibrations of a damped structure can be represented by inserting modal damping terms into the above uncoupled equation as follows:

$$\ddot{g}_{mn} + \eta_{mn}\omega_{mn}\dot{g}_{mn} + \omega_{mn}^2 g_{mn} = \frac{1}{M_{mn}} X_m(x_0)\,Y_n(y_0)\,F(t) \quad (m, n = 1, 2, 3, \cdots) \quad (13)$$

where η_{mn}, loss factor of the mn-th mode, is defined by

$$\eta_{mn} = \Big(\frac{V_I^{edge}}{V^{plate} + V_R^{edge}}\Big)_{mn} \quad (14)$$

In Eq. (14), V^{plate} is the modal strain energy of the plate, and V_R^{edge} and V_I^{edge} respectively denote the modal strain energy at the boundary support due to real and imaginary parts of the complex stiffness at the edge and they have the following forms as given by Kang (1996):

$$(V^{plate})_{mn} = \frac{D}{2}\int_0^a \int_0^b \Big[\Big(\frac{\partial^2 W_{mn}}{\partial x^2}\Big)^2 + \Big(\frac{\partial^2 W_{mn}}{\partial y^2}\Big)^2 + 2\nu \frac{\partial^2 W_{mn}}{\partial x^2}\frac{\partial^2 W_{mn}}{\partial y^2}$$

$$+2(1-\nu)\left(\frac{\partial^2 W_{mn}}{\partial x \partial y}\right)^2\Bigg]dxdy \quad (15\text{-a})$$

$$\begin{aligned}(V_R^{edge})_{mn}=&\frac{\mathrm{Re}[K_{11}]}{2}\left\{\int_0^a[W_{mn}^2(x,0)\right.\\&+W_{mn}^2(x,b)]dx+\int_0^b[W_{mn}^2(0,y)\\&\left.+W_{mn}^2(a,y)]dy\right\}\\&+\frac{\mathrm{Re}[K_{22}]}{2}\left\{\int_0^a\left[\left(\frac{\partial W_{mn}(x,0)}{\partial y}\right)^2\right.\right.\\&\left.+\left(\frac{\partial W_{mn}(x,b)}{\partial y}\right)^2\right]dx\\&+\int_0^b\left[\left(\frac{\partial W_{mn}(0,y)}{\partial x}\right)^2\right.\\&\left.\left.+\left(\frac{\partial W_{mn}(a,y)}{\partial x}\right)^2\right]dy\right\}\end{aligned} \quad (15\text{-b})$$

$$\begin{aligned}(V_I^{edge})_{mn}=&\frac{\mathrm{Im}[K_{11}]}{2}\left\{\int_0^a[W_{mn}^2(x,0)\right.\\&+W_{mn}^2(x,b)]dx+\int_0^b[W_{mn}^2(0,y)\\&\left.+W_{mn}^2(a,y)]dy\right\}\\&+\frac{\mathrm{Im}[K_{22}]}{2}\left\{\int_0^a\left[\left(\frac{\partial W_{mn}(x,0)}{\partial y}\right)^2\right.\right.\\&\left.+\left(\frac{\partial W_{mn}(x,b)}{\partial y}\right)^2\right]dx\\&+\int_0^b\left[\left(\frac{\partial W_{mn}(0,y)}{\partial x}\right)^2\right.\\&\left.\left.+\left(\frac{\partial W_{mn}(a,y)}{\partial x}\right)^2\right]dy\right\}\end{aligned} \quad (15\text{-c})$$

From Eq. (4), Eq. (5), and orthogonality of the undamped modes, initial conditions for $g_{mn}(t)$ are given by:

$$g_{mn}(0)=0 \text{ and } \dot{g}_{mn}(0)=0 \quad (16)$$

Solving Eq. (13) together with Eq. (15) using the Duhamel integral, $g_{mn}(t)$ can be obtained as

$$g_{mn}(t)=\frac{X_m(x_0)\,Y_n(y_0)}{M_{mn}\bar{\omega}_{mn}}\int_0^t F(s)$$
$$\exp[-\xi_{mn}\omega_{mn}(t-s)]\sin[\bar{\omega}_{mn}(t-s)]ds \quad (17)$$

where the damping ratio ξ_{mn} and damped natural frequency $\bar{\omega}_{mn}$ are defined respectively by

$$\xi_{mn}=\frac{\eta_{mn}}{2},\quad \bar{\omega}_{mn}=\sqrt{1-\xi_{mn}^2}\cdot\omega_{mn} \quad (18)$$

Substituting Eq. (17) into Eq. (5) yields displacement and acceleration responses of the plate as follows:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{X_m(x_0)\,Y_n(y_0)}{M_{mn}\bar{\omega}_{mn}}X_m(x)\,Y_n(y)\cdot$$
$$\int_0^t F(s)\exp[-\xi_{mn}\omega_{mn}(t-s)]\cdot$$
$$\sin[\bar{\omega}_{mn}(t-s)]ds \quad (19\text{-a})$$

$$\begin{aligned}\ddot{w}(x,y,t)=&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{X_m(x_0)\,Y_n(y_0)}{M_{mn}}X_m(x)\,Y_n(y)\cdot\\&\left\{F(t)+\int_0^t F(s)\exp(-\xi_{mn}\omega_{mn}(t-s))\right.\\&\times\left[\frac{2\xi_{mn}^2-1}{\sqrt{1-\xi_{mn}^2}}\omega_{mn}\sin(\bar{\omega}_{mn}(t-s))\right.\\&\left.\left.-2\xi_{mn}\omega_{mn}\cos(\bar{\omega}_{mn}(t-s))\right]ds\right\}\end{aligned} \quad (19\text{-b})$$

Linear spectra of the acceleration in the frequency domain is given by

$$-\omega^2 W(x,y,j\omega)=-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{X_m(x_0)\,Y_n(y_0)}{M_{mn}\omega_{mn}^2}\cdot$$
$$X_m(x)\,Y_n(y)\frac{\omega^2 F(j\omega)}{[1-(\omega/\omega_{mn})^2+j\eta_{mn}(\omega/\omega_{mn})]} \quad (20)$$

where $W(x,y,j\omega)$ and $F(j\omega)$ are the Fourier transform of the $w(x,y,t)$ and $F(t)$, respectively.

2.4 Acoustic radiation

Pressure of the sound radiated from a vibrating plate in an infinite baffle can be obtained by evaluating the Rayleigh surface integral (Rayleigh, 1945), in which areal elements of the plate are regarded as simple point sources of outgoing waves and the sound pressure level at a point (d, Ψ, θ) in space as shown in Fig. 2 is given by

$$p(d,\Psi,\theta,t)=\frac{\rho_0}{2\pi}\int_0^a\int_0^b\ddot{w}\left\{x,y,t-\frac{R(x,y)}{c}\right\}\cdot$$
$$\frac{1}{R(x,y)}dxdy \quad (21)$$

where ρ_0 and c are mass density and wave veloc-

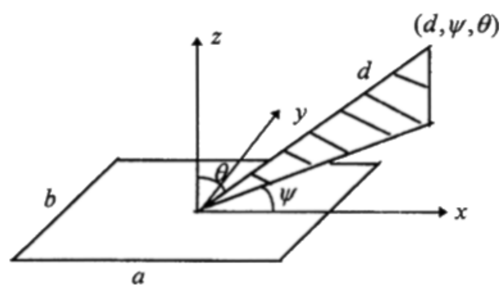


Fig. 2 Coordinate system for sound pressure estimation

ity of the acoustic medium respectively, $\ddot{w}(x, y, t)$ the acceleration-time history of the plate vibration, and $R(x, y)$ the distance from the receiver to a point (x, y) on the plate surface.

Linear frequency spectra of the sound pressure is given by

$$P(d, \Psi, \theta, j\omega) = -\frac{\rho_0}{2\pi}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{X_m(x_0)\,Y_n(y_0)}{M_{mn}\omega_{mn}^2}\cdot \int_0^a\int_0^b \frac{X_m Y_n \exp(-j\omega R/c)}{d}dxdy \cdot \frac{\omega^2 F(j\omega)}{[1-(\omega/\omega_{mn})^2 + j\eta_{mn}(\omega/\omega_{mn})]} \quad (22)$$

3. Results and Discussion

Vibration response and consequent sound radiation of a thin square plate subject to an impact at the plate center is computed numerically in the time and frequency domain for several widths of the viscoelastic boundary supports along the four edges. Geometry and material parameters of the plate are: $a=200$ mm, $h=1.6$ mm, $E=210,000$ MPa, $\eta=0.01$, $\nu=0.3$, $\rho=7,850$ Kg/m^3. Thickness and width of the viscoelastic boundary supports are: $H=0.254$ mm and $L=4$, 8, 16, and 32 mm, and the material properties are quoted from Kang(1996), which are modeled as springs with complex stiffness as in Kang(1996). Shape of the pulse used for the impact is a rectangle with duration of $T_f=0.1$ ms and magnitude of $F=700N$.

Time domain acceleration response at center of the plate and sound pressure at a point on the z-axis 0.4m apart from the plate are shown in Figs. 3 and 4 respectively. Figures 5 and 6 show the results in frequency domain. Figure 7 shows variation of the peak sound pressure level with the viscoelastic support width for various vibration modes. It can be seen from these results that there exists an optimum support width around 8

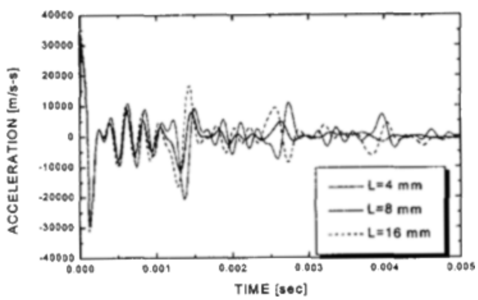


Fig. 3 Acceleration response in time domain of the plate at the impact point for three different widths of the viscoelastic boundary support

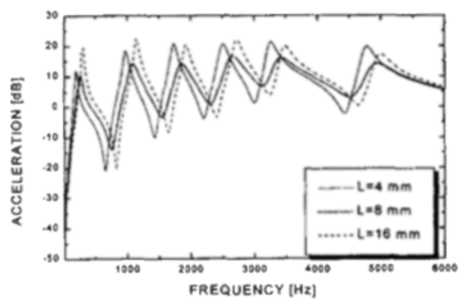


Fig. 5 Acceleration spectra of the plate at the impact point for three different widths of the viscoelastic boundary support

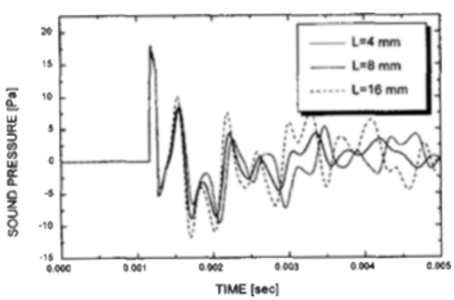


Fig. 4 Sound pressure level in time domain at a point on the z-axis($z=0.4$ m) for three different widths of the viscoelastic boundary support

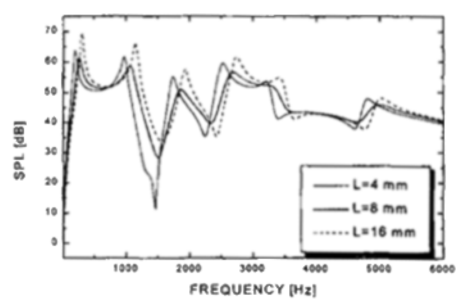


Fig. 6 Sound pressure level in frequency domain at a point on the z-axis($z=0.4$ m) for three different widths of the viscoelastic boundary support

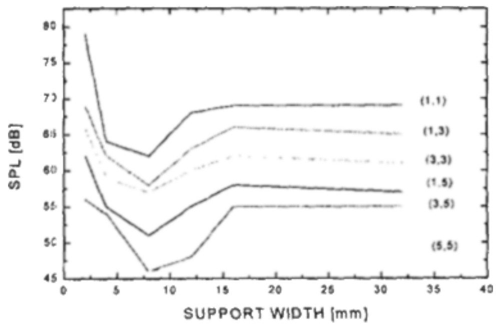


Fig. 7 Variation of the peak sound pressure level with width of support

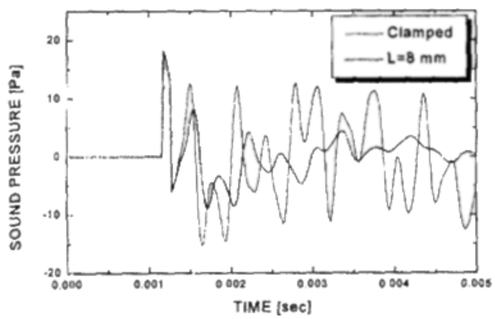


Fig. 8 Comparison of sound pressure response in time domain at a point between viscoelastic and clamped boundary supports

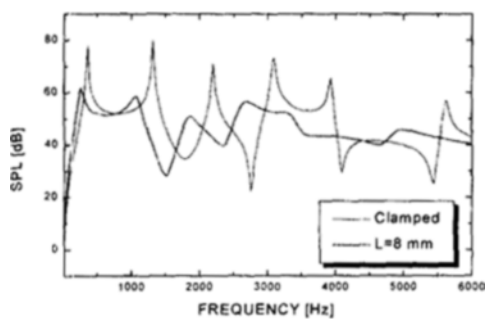


Fig. 9 Comparison of sound pressure response in frequency domain at a point $z=0.4$ mm between ciscoelastic and clamped boundary supports

mm at which the sound radiation is minimized. Figure 8 compares the sound pressure from plates with viscoelastic boundary supports (width $L=8$ mm) with the one under fixed boundary condition in time domain. Figure 9 shows the corresponding sound pressure levels in frequency domain, where it can be seen that roughly 20 dB can be lowered by an appropriate width of viscoelastic boundary support.

4. Conclusion

Effects of viscoelastic boundary support widths on the vibration as well as radiation of sound has been analyzed in the time and frequency domain for a square plate subject to an impact loading. Damped responses have been obtained by introducing modal damping defined by the modal strain energy method. It is shown that viscoelastic treatment at the boundary support is a very practical tool for reduction of the vibration and sound radiation, and that width of the viscoelastic support must be adjusted to maximize the effectiveness of the treatment.

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